

# Electric dipole moments of charged leptons with sterile fermions

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## Abstract

We address the impact of sterile fermions on charged lepton electric dipole moments. We show that in order to have a non-vanishing contribution to electric dipole moments, the minimal extension necessitates the addition of at least two sterile fermion states. Sterile neutrinos can give significant contributions to the charged lepton electric dipole moments if the masses of the non-degenerate sterile states are both above the electroweak scale. In addition, the Majorana nature of neutrinos is also important. Furthermore, we apply the computations of the electric dipole moments for the most minimal realisation of the Inverse Seesaw mechanism, in which the Standard Model is extended by two right-handed neutrinos and two sterile fermion states. We show that the two pairs of (heavy) pseudo-Dirac mass eigenstates can give significant contributions to the electron electric dipole moment, lying close to future experimental sensitivity. We further discuss the possibility of beyond the minimal Inverse Seesaw models and of having a successful leptogenesis in this framework.

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# 1 Introduction

Electric Dipole Moments (EDMs) are CP violating observables which are sensitive to new physics. In the Standard Model (SM), the Cabibbo-Kobayashi-Maskawa matrix is the only source of CP violation, and the charged lepton EDMs are induced at four-loop level. The corresponding order of magnitude is roughly given by

$$|d_e|/e \sim \frac{\alpha_W^3 \alpha_s m_e}{246(4\pi)^4 m_W^2} J_{CP} \sim 10^{-38} \text{ cm}, \quad (1)$$

where  $J_{CP} \equiv \text{Im}(V_{us}V_{cs}^*V_{cb}V_{ub}^*)$  is the Jarlskog invariant whose value is fixed to be  $J_{CP} \approx 10^{-5}$  by the experiments [1]. For the other charged leptons, muon and tau, the EDMs in the SM are estimated by replacing the electron mass  $m_e$  in Eq. (1) by the mass of muon or tau. On the other hand, the current bounds of the charged lepton EDMs are given by

$$|d_e|/e < 8.7 \times 10^{-29} \text{ cm} \quad (\text{ACME Collaboration}) [2], \quad (2)$$

$$|d_\mu|/e < 1.9 \times 10^{-19} \text{ cm} \quad (\text{Muon } g-2 \text{ Collaboration}) [3], \quad (3)$$

$$|d_\tau|/e < 4.5 \times 10^{-17} \text{ cm} \quad (\text{Belle Collaboration}) [4]. \quad (4)$$

Thus the SM predictions of the charged lepton EDMs are far below the current experimental bounds. However if one considers some extensions of the SM, new contributions may be able to generate larger values close to the current experimental bounds or future sensitivities.

There are a number of extensions of the SM to address some of its observational caveats such as non-zero neutrino masses, existence of dark matter and baryon asymmetry in the Universe. Adding sterile fermions to the SM is one of the most economical extensions of the SM. We consider models extended with only sterile fermions, and compute the charged lepton EDMs. First, we do not impose a specific seesaw mechanism for neutrino masses like Type-I Seesaw, Inverse Seesaw or Linear Seesaw mechanism (the effective model). Second, the minimal Inverse Seesaw model is considered as a more specific case.

## 2 The Effective Model

The SM is extended with  $N$  sterile fermions. The relevant interactions in the Feynman-'t Hooft gauge are given by

$$\mathcal{L} = -\frac{g_2}{\sqrt{2}} U_{\alpha i} W_\mu^- \bar{\ell}_\alpha \gamma^\mu P_L \nu_i - \frac{g_2}{\sqrt{2}} H^- \bar{\ell}_\alpha \left( \frac{m_\alpha}{m_W} P_L - \frac{m_i}{m_W} P_R \right) \nu_i + \text{H.c.}, \quad (5)$$

where  $H^-$  is the Goldstone boson absorbed by the  $W$  gauge boson,  $U_{\alpha i}$  is the mixing matrix of the neutrinos ( $\nu_\alpha = U_{\alpha i} \nu_i$ ),  $\ell_\alpha$  is the charged lepton and  $\nu_i$  is the mass eigenstates of the neutrinos. The indices denote  $\alpha = e, \mu, \tau$  and  $i, j = 1, \dots, N$ .

We do not fix any neutrino mass generation mechanism and the neutrino masses  $m_i$ , and thus the mixing matrix elements  $U_{\alpha i}$  are regarded as independent parameters. The mixing matrix  $U_{\alpha i}$  in Eq. (5) is the sub-matrix of the whole mixing matrix  $U_{ij}$  which is appropriately parametrized.

The leading contribution to the charged lepton EDMs is induced at two-loop level via the diagrams shown in Fig. 1, which includes a total of 44 diagrams in the Feynman-'t Hooft gauge.

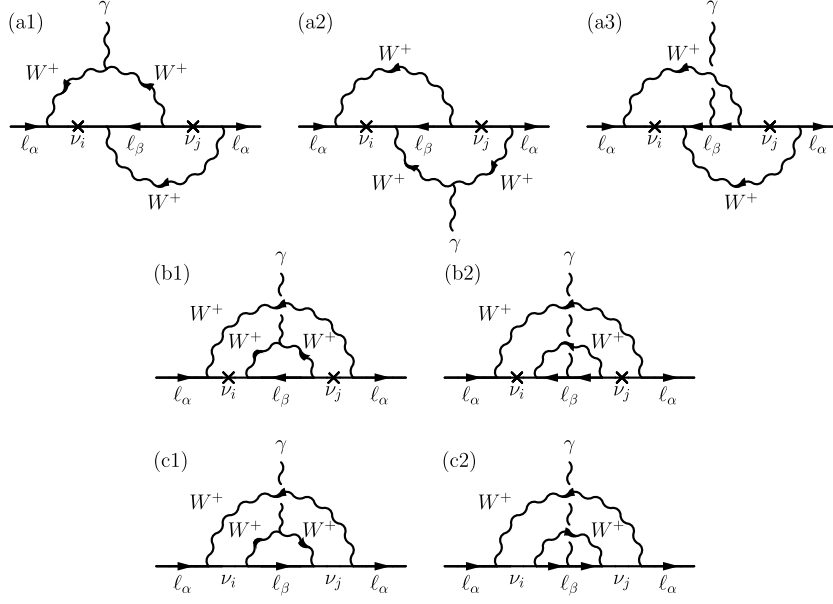


Figure 1: Diagrams contributing to charged lepton EDMs.

We compute all the diagrams using FeynCalc [5] in order to obtain an analytic result which is given in the Appendix of the reference [6]. The charged lepton EDMs are formally written as

$$d_\alpha = -\frac{g_2^4 e m_\alpha}{4(4\pi)^4 m_W^2} \sum_\beta \sum_{i,j} \sqrt{x_i x_j} \left[ J_{ij\alpha\beta}^M I_M(x_i, x_j) + J_{ij\alpha\beta}^D I_D(x_i, x_j) \right], \quad (6)$$

where  $x_i \equiv m_i^2/m_W^2$ ,  $J_{ij\alpha\beta}^M$  and  $J_{ij\alpha\beta}^D$  are the phase factors of the Majorana type and Dirac type contributions defined by  $J_{ij\alpha\beta}^M \equiv \text{Im} \left( U_{\alpha j} U_{\beta j} U_{\beta i}^* U_{\alpha i}^* \right)$  and  $J_{ij\alpha\beta}^D \equiv \text{Im} \left( U_{\alpha j} U_{\beta j}^* U_{\beta i} U_{\alpha i}^* \right)$ , and the charged lepton masses can be safely neglected compared to the  $W$  gauge boson mass  $m_\alpha^2 \ll m_W^2$ . From the definition of the phase factors  $J_{ij\alpha\beta}^M$  and  $J_{ij\alpha\beta}^D$ , one can see that they are anti-symmetric under the exchange  $i \leftrightarrow j$ . As a result, one can verify that only the anti-symmetric part of the loop functions  $I_M$  and  $I_D$  can contribute to the EDMs in Eq. (6).

In the case of  $N = 1$ , the EDM expression can be simplified as [6]

$$d_\alpha \approx -\frac{g_2^4 e m_\alpha}{2(4\pi)^4 m_W^2} \sum_\beta \sum_{i=1}^3 \sqrt{x_i x_4} J_{i4\alpha\beta}^D I_D(0, x_4). \quad (7)$$

Therefore, taking  $I_D(0, x_4) \sim 1$  and  $x_i \sim 10^{-24}$  ( $i = 1, 2, 3$ ), the predicted electron EDM is evaluated to be  $|d_e|/e \lesssim 10^{-39}$  cm, which is too small compared to the current experimental bound. For  $N = 2$ , the EDM expression can be written as

$$d_\alpha \approx -\frac{g_2^4 e m_\alpha}{2(4\pi)^4 m_W^2} \sqrt{x_4 x_5} \left[ J_\alpha^M I_M(x_4, x_5) + J_\alpha^D I_D(x_4, x_5) \right], \quad (8)$$

where  $J_\alpha^{M/D} = \sum_\beta J_{45\alpha\beta}^{M/D}$ . Only heavy sterile states in the loop provide a dominant contribution, and the EDMs can be potentially large enough to be detected. Two different sterile states enter

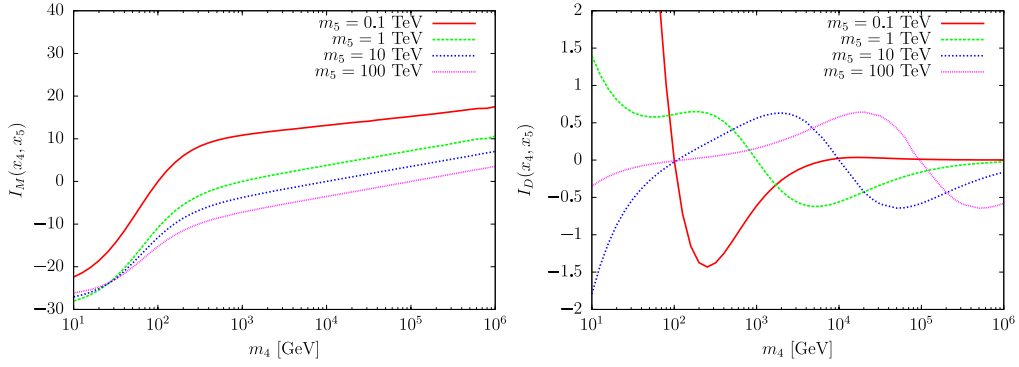


Figure 2: Loop functions  $I_D$  and  $I_M$  as a function of  $m_4$  for fixed  $m_5$ .

in the loops as shown in Fig. 1. Thus one can see that at least two sterile fermions are needed to induce sizeable EDMs. If  $N$  sterile fermions are added to the SM, the predicted EDMs are expected to increase with the factor  $N(N-1)/2$ .

The behavior of the loop functions is shown in Fig. 2, where several values for  $m_5$  are chosen. From the plots, one can see that  $I_M \gg I_D$  for  $x_4, x_5 \gg 1$ . Although the loop function  $I_D$  can be larger than  $I_M$  if  $x_4, x_5 \ll 1$ , the predicted EDMs are very suppressed since the loop functions can be expanded in  $x_4$  and  $x_5$  as  $I_{M/D}(x_4, x_5) \propto (x_4 - x_5) \ll 1$ , due to the anti-symmetry.

The experimental and theoretical constraints are taken into account: these include neutrino oscillation data, charged lepton flavour violation, direct production of the sterile neutrinos at colliders, electroweak precision tests including  $W$  boson decay,  $Z$  invisible decay, lepton flavour universality of meson decays, non-unitarity of the mixing matrix  $U_{\alpha i}$  and perturbative unitarity bound [8]. With these constraints, the electron EDM is computed as shown in Fig. 3. The red points are excluded by the charged lepton flavour violation bounds, while the green points are allowed by all the above experimental and theoretical constraints. The blue dotted line is the current bound of the electron EDM as measured by the ACME collaboration, and the black dotted line shows the future sensitivity of the upgraded ACME collaboration. The region of  $m_4 \lesssim \mathcal{O}(100)$  GeV is excluded by the electroweak precision tests. The upper region is mainly constrained by the charged lepton flavour violation bounds and the upper right region is excluded by perturbative unitarity bound. All the green points are below the current bound of the electron EDM, however some points can be within reach of the future sensitivity  $|d_e|/e = 10^{-30}$  cm. Figure 4 shows the allowed parameter space from all the constraints in the  $(m_i - |U_{ei}|^2)$  plane. The colored regions are excluded. The green points represent the parameter space inducing an electron EDM larger than the future sensitivity  $|d_e|/e \gtrsim 10^{-30}$  cm. One can see that some green points can also be testable by a future ILC experiment (violet line). The EDMs for the other charged leptons (muon and tau) are also computed and compared to the current bounds and future sensitivities. However the order of the magnitude for the predictions is too small to be detected in near future.

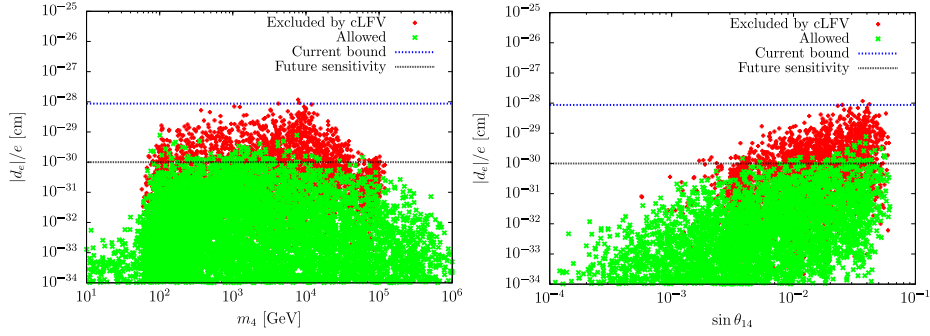


Figure 3: Predicted electron EDM with the experimental and theoretical constraints. The red points are excluded by the charged lepton flavour violation bounds while the green points are allowed by all the constraints.

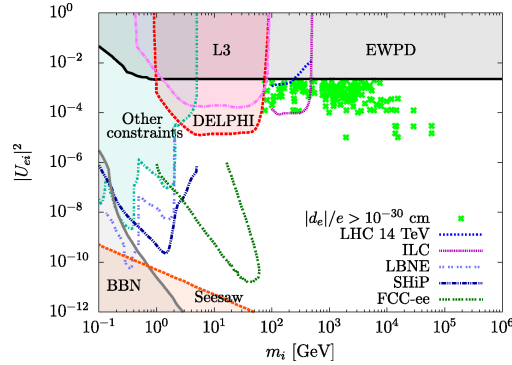


Figure 4: Allowed parameter space in the  $(m_i-|U_{ei}|^2)$  plane. The colored regions are excluded. The green points can induce an electron EDM larger than the future sensitivity  $|d_e|/e = 10^{-30}$  cm.

### 3 The Inverse Seesaw Models

We consider the Inverse Seesaw as a specific model. The SM is extended with both right-handed neutrinos  $N^c$  and singlet fermions  $s$ . At least two pairs of these are required to accommodate neutrino oscillation data. This minimal Inverse Seesaw model is denoted as (2, 2)-model [9]. In the minimal Inverse Seesaw model, the neutrino mass matrix is given by

$$M = \begin{pmatrix} 0 & 0 & 0 & d_{11} & d_{12} & 0 & 0 \\ 0 & 0 & 0 & d_{21} & d_{22} & 0 & 0 \\ 0 & 0 & 0 & d_{31} & d_{32} & 0 & 0 \\ d_{11} & d_{21} & d_{31} & m_{11} & m_{12} & n_{11} & n_{12} \\ d_{12} & d_{22} & d_{32} & m_{21} & m_{22} & n_{12} & n_{22} \\ 0 & 0 & 0 & n_{11} & n_{12} & \mu_{11} & \mu_{12} \\ 0 & 0 & 0 & n_{12} & n_{22} & \mu_{12} & \mu_{22} \end{pmatrix}, \quad (9)$$

in the basis  $(\nu_e, \nu_\mu, \nu_\tau, N_1^c, N_2^c, s_1, s_2)^T$ . The matrix element  $d_{ij}$  is the Dirac mass term between the left-handed and the right-handed neutrinos given by electroweak symmetry breaking, and  $n_{ij}$

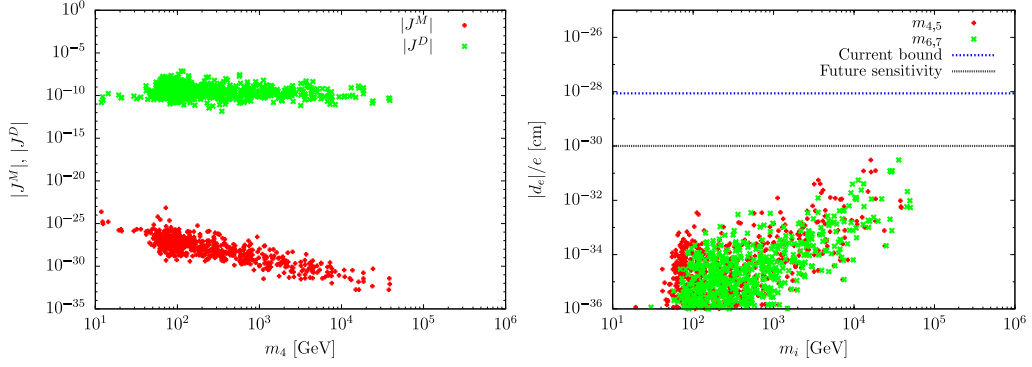


Figure 5: Charged lepton EDMs in the (2, 2) Inverse Seesaw model.

is the Dirac mass term between the right-handed neutrinos  $N^c$  and the singlet fermions  $s$ . The matrix elements  $m_{ij}$  and  $\mu_{ij}$  are Majorana mass terms violating lepton number conservation. These Majorana mass terms are (naturally) assumed to be much smaller than the other Dirac mass terms. The mass matrix  $M$  can be diagonalized as  $U^T M U = \text{diag}(m_1, m_2, \dots, m_7)$  with the mass hierarchy  $|\mu|, |m| \ll |d| \ll |n|$ . The mass eigenvalues  $m_{1,2,3}$  correspond to the active light neutrino masses. The heavy mostly sterile states turn out to be nearly degenerate as  $m_4 \approx m_5$  and  $m_6 \approx m_7$  due to the nature of Inverse Seesaw models. This mass degeneracy means that the heavy sterile states construct pseudo-Dirac fermions.

The relevant diagrams to the charged lepton EDMs in the minimal Inverse Seesaw model are the same as those of the effective model shown in Fig. 1. However since the heavy sterile states are pseudo-Dirac fermions with nearly degenerate masses, the Majorana type contribution is negligible. Thus, one can focus on the Dirac type diagrams (c1) and (c2) in Fig. 1, and express the EDM formula as

$$d_\alpha \approx -\frac{g_2^4 e m_\alpha}{2(4\pi)^4 m_W^2} J^D I'_D(x_4, x_6), \quad (10)$$

where  $I'_D(x_4, x_6)$  is the reduced loop function whose analytic formula and numerical evaluations are given in the reference [7].

The numerical computations for the phase factors  $|J^M|$  and  $|J^D|$ , and the predicted electron EDM are shown in Fig. 5 where all the relevant constraints which have been considered in the effective model are imposed. From the left plot of Fig. 5, one can see that  $|J^D|$  is much larger than  $|J^M|$ , as expected because of the pseudo-Dirac heavy sterile states. The right plot shows that the maximum value of the predicted electron EDM is slightly below the future prospects,  $|d_e|/e = 10^{-30}$  cm. Similar to the effective model, the other charged lepton (muon and tau) EDMs have also been computed, but the predictions are very far from the current bounds and future prospects.

In the above, we considered the minimal Inverse Seesaw model with two pairs of the right-handed neutrinos and singlet sterile fermions. If  $N > 2$  pairs are added to the SM, the electron EDM is roughly given by

$$d_e^{(N,N)} \sim \frac{g_2^4 e m_e}{2(4\pi)^4 m_W^2} |4N(N-1) \text{Im}(U^4) I'_D|. \quad (11)$$

Thus one can see that the electron EDM is enhanced if more sterile fermions are introduced. However the constraints from charged lepton flavour violation also become stronger at the same time. The most stringent process is  $\mu \rightarrow e\gamma$  whose branching ratio can be roughly given by

$$\text{Br}(\mu \rightarrow e\gamma) \sim \frac{\sqrt{2}G_F^2 m_\mu^5}{\Gamma_\mu} (2N)^2 |U|^4 \leq 4.2 \times 10^{-13}. \quad (12)$$

Combining Eq. (11) and (12), the enhancement factor compared to the minimal (2,2)-model is given by

$$\left| \frac{d_e^{(N,N)}}{d_e^{(2,2)}} \right| \lesssim 2 \left( 1 - \frac{1}{N} \right). \quad (13)$$

Therefore one can see that the maximum enhancement factor is 2 from this rough estimation.

We may also have a correlation with another CP violating observable. In particular, the baryon asymmetry of the Universe may be generated by resonant leptogenesis since the heavy sterile states are naturally nearly degenerate in Inverse Seesaw models. In order to generate a sufficient baryon asymmetry of the Universe, the out-of-equilibrium condition  $\Gamma_{\nu_4} < H(T)|_{T=m_4}$  should be satisfied for the lightest heavy sterile state, where  $H(T)$  is the Hubble parameter and the decay rate  $\Gamma_{\nu_4}$  is given by

$$\Gamma_{\nu_4} = \frac{g_2^2 m_4^3}{16\pi m_W^2} \sum_\alpha |U_{\alpha 4}|^2. \quad (14)$$

Thus, the following condition can be derived

$$\sum_\alpha |U_{\alpha 4}|^2 \lesssim 10^{-15} \left( \frac{1 \text{ TeV}}{m_4} \right) \left( \frac{g_*}{100} \right)^{1/2}, \quad (15)$$

with  $g_*$  the effective degrees of freedom of relativistic particles. One can verify that the order of magnitude of the mixing matrix  $|U_{\alpha 4}|$  required to generate the baryon asymmetry of the Universe is much smaller than what would be required to obtain a large electron EDM (see Fig. 4). If the mixing matrix is fixed as large value, as  $|U_{\alpha 4}|^2 \sim 10^{-3}$  as to obtain a large electron EDM, only a small baryon asymmetry would be generated due to the strong washout effect.

## 4 Summary

We have computed the charged lepton EDMs at two-loop level in the effective model and in the minimal Inverse Seesaw model. We found that at least two sterile fermions are needed to obtain a sizeable electron EDM, within sensitivity reach of the upgraded ACME future experiment. In Inverse Seesaw models, the Dirac type contribution is dominant since the pairs of heavy sterile fermions construct pseudo-Dirac fermions. In this case, the maximum value of the electron EDM is slightly below the future sensitivity. The heavy sterile fermion masses should be in the range of 100 GeV – 10 TeV to induce large charged lepton EDMs due to experimental and theoretical constraints.

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